(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 2168 **Roll No.** | | | | | | | | | | |

B. Tech.

(SEM. V) THEORY EXAMINATION 2011-12

GRAPH THEORY

Time : 2 Hours *Total Marks : 50*

Note :– (i) Attempt **all** questions.

(ii) Make suitable assumptions wherever necessary.

(iii) Notions/symbols used have usual meaning.

1. Attempt any **four** parts of the following : **(3×4=12)**

(a) Let $n \geq 4$ be any even number. Show by induction that there exists a 3-regular graph G with $v(G) = n$.

(b) Find all nonisomorphic simple graphs of order 4.

(c) Define the following operations on the graphs with example :–

(i) Product

(ii) Complement

(iii) Ring sum.

(d) Let G be a disconnected graph of order 5. What is the largest possible value for e(G) ? If G is a disconnected graph of order $n \geq 2$, what is the largest possible value for e(G) ? Construct one such extremal graph of order n.

(e) Suppose G and G' are two graphs having n vertices. For what values of n is it possible for G to have more components and edges than G′ ?

(f) Show that any circuit in a graph contains a cycle.

2. Attempt any **two** parts of the following : **(6×2=12)**

(a) Show that :

(i) Any connected graph with n vertices and n–1 edges is a tree.

(ii) In any tree (with two or more vertices), there are at least two pendant vertices.

(b) Define the term metric and associated number of a graph. Show every tree has either one or two centers.

(c) Write the Kruskal's algorithm for finding the minimum spanning tree of a graph. Discuss its performance.

3. Attempt any **two** parts of the following : **(6×2=12)**

(a) Define the cut sets and cut vertices of a graph. Prove that in a nonseparable graph G the set of edges incident on each vertex of G is a cut set.

(b) Using the geometric arguments prove that the Kuratowski's second graph is nonplanar.

(c) (i) Determine the number of crossings and thickness of the graph K_5.

(ii) Show that the thickness of the eight vertex complete graph is two, where as that of the nine vertex complete graph is three.

4. Attempt any **four** parts of the following : **(3.5×4=14)**

(a) Prove that the set consisting of all the cut-sets and the edge-disjoint union of cut-sets (including the null set) in a graph G is an abelian group under the ring-sum operation.

(b) Explore how the covering number of a graph G with n vertices is related to the diameter of G.

(c) What is it meant by the Basis Vectors of a graph ? Explain with an example.

(d) Show that a complete matching of V_1 into V_2 in a bipartite graph exists if and only if every subset of **r** vertices in V_1 is collectively adjacent to **r** or more vertices in V_2 for all values of **r**.

(e) Define the incidence matrix of a connected graph with n vertices and e edges and prove that rank of incidence matrix of the graph is n-1.

(f) Find chromatic polynomial P(G, x), where G is a cyclic graph with n vertices where n = 3 or n = 4.